mated amount of bills payable, \$2,170, there results an estimated final balance on 1921 business of approximately \$1,530. The corresponding estimated final balance one year ago on 1920 business was \$1,360. The apparently better showing of \$170 in 1921 over 1920 is more than accounted for by the "special contributions" of \$202.60 in 1921; thus the Association *has just been able to match expenditures with income.* It is only by the most careful attention to the affairs of the Association, by soliciting additional advertising, by repeated appeals to part of our members to pay their dues, and by similar time-consuming efforts, that the officers are able to make this showing. A greater margin of safety must be attained and this can be done through the coöperation of all members, especially in seeking steadily for new members.

W. D. CAIRNS, *Secretary-Treasurer.*

SERRET'S ANALOGUE OF DESARGUES'S THEOREM.

By H. S. WHITE, Vassar College.

In the course of a discussion of the eight points that lie on three quadric surfaces but not on any gauche cubic curve, P. Serret announced (*Géométrie de Direction,* 1869, page 325) an extension of Desargues's theorem concerning a quadrangle inscribed in a conic. This extension states an involutional property of a complete hexagon whose six vertices lie on a gauche cubic. Such a hexagon has 20 plane faces, or 10 pairs of opposite plane faces, each pair containing all six vertices. Any chord of the gauche cubic meets the cubic in points P and Q, and is cut by the 20 planes in 10 pairs of points. The theorem is: *The 10 pairs of points lie in a quadric involution, and P, Q form an 11th pair in the same involution.*

The proof, being inclusive of other things, is unnecessarily long for this particular purpose, and it is worth while to notice that the ordinary synthetic proof of the theorem on conics can be applied with very little change to this extension. Indeed, the only change required is to use pencils of planes through two chords of a cubic curve in place of the flat pencils of lines through two points of a conic.

Let P and Q denote two points on a gauche cubic curve. Six other points shall be called A, A_1, B, B_1, C, C_1. Consider the curve as generated by three projective pencils of planes, two of which have for axes the chords AA_1 and BB_1 respectively. Four pairs of corresponding planes will be

$$\begin{Bmatrix} AA_1P \\ BB_1P \end{Bmatrix}, \begin{Bmatrix} AA_1Q \\ BB_1Q \end{Bmatrix}, \begin{Bmatrix} AA_1C \\ BB_1C \end{Bmatrix}, \begin{Bmatrix} AA_1C_1 \\ BB_1C_1 \end{Bmatrix}.$$

Permute the four planes through BB_1 from the order $PQCC_1$ to the projective order QPC_1C. Then take sections of the two sets of planes by the chord PQ.

On this chord are found two projective quadruples of points:

$$\begin{array}{llll} P,\ Q, & \text{one in } AA_1C, & \text{one in } AA_1C_1; \\ Q,\ P, & \text{" " } BB_1C_1, & \text{" " } BB_1C. \end{array}$$

Owing to the double correspondence of P and Q, these are three pairs in involution. Aside from P and Q, two points of a pair are found in two planes determined by mutually exclusive triads taken from the given 6 points.

As any two chords can be chosen for axes, we may use AA_1 and B_1C_1. The resulting tetrads of points on PQ are then:

$$\begin{array}{llll} P,\ Q, & \text{one in } AA_1C, & \text{one in } AA_1B; \\ Q,\ P, & \text{" " } B_1C_1B, & \text{" " } B_1C_1C. \end{array}$$

From the first three pairs it is found that this involution is identical with the former, hence the fourth pair lies in the same involution. That fourth pair, in planes AA_1B and CC_1B_1, is determined like the former pairs by two mutually exclusive (or supplementary) triads of points in the given sextette. By repetition of this permutation process we can show that any desired pair of supplementary triads give, on the chord PQ, a pair in this same involution. In all, there are ten such pairs of planes, giving ten pairs of points in involution in addition to the pair P, Q.

As Serret points out, it is sufficient to show that the eight faces of a simple octahedron on the six points cut a line in four pairs of points in involution, and from this can be inferred the remainder. Hence there may be written down two equations in line-coördinates which are satisfied by chords (double secants) of a gauche cubic through the six vertices of the octahedron.

A CERTAIN TWO-DIMENSIONAL LOCUS.

By J. L. WALSH, Harvard University.

The writer has recently had occasion to consider the following problem, in connection with the approximate determination of the roots of certain types of polynomials:[1] If two points z_1 and z_2 have as their respective loci the interiors (boundaries included) of two circles, determine the locus of a point z given by the relation $z = (m_2z_1 + m_1z_2)/(m_1 + m_2)$, when m_1 and m_2 are real or complex constants. A closely allied problem is found by considering the locus of the point z determined as before, but where in addition m_1 and m_2 also vary, and take all

[1] See several papers already published and others about to be published in the *Transactions of the American Mathematical Society*.

The following interpretation can be given to the problem of the present note: Let the points $\alpha_1, \alpha_2, \cdots \alpha_n$ vary independently and have as common locus the interior and boundary of a circle C_1, and let the points $\beta_1, \beta_2, \cdots \beta_n$ vary independently and have as common locus the interior and boundary of a circle C_2; determine the locus of the roots of the equation

$$(z - \alpha_1)(z - \alpha_2) \cdots (z - \alpha_n) = A(z - \beta_1)(z - \beta_2) \cdots (z - \beta_n)$$

where A takes all the values such that $|A| = 1$.